

① Matrix :-

Matrix is a rectangular array of numbers enclosed by a pair of brackets $[]$ or $()$, subject to certain rules of operation. A matrix is a rectangular arrangement of numbers in rows & columns. General form of matrix can be written :-

$$A = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & \dots & C_n \end{matrix} \\ \begin{matrix} a_{11} \\ a_{21} \\ a_{31} \\ \dots \\ a_{m1} \end{matrix} & \begin{matrix} a_{12} \\ a_{22} \\ a_{32} \\ \dots \\ a_{m2} \end{matrix} & \begin{matrix} a_{13} \\ a_{23} \\ a_{33} \\ \dots \\ a_{m3} \end{matrix} & \dots & \begin{matrix} a_{1n} \\ a_{2n} \\ a_{3n} \\ \dots \\ a_{mn} \end{matrix} \\ \begin{matrix} R_1 \\ R_2 \\ R_3 \\ \dots \\ R_m \end{matrix} \end{matrix}$$

⇒ Types of Matrix

1. Rectangular Array ⇒ A matrix in which no. of rows & columns are not equal to number of columns. It means $m \neq n$.

Ex.

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix}_{2 \times 3}$$

2) Square Matrix \rightarrow In which number of rows are equal to number of columns, means $m = n$.

Ex.

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 0 \\ 1 & 3 & 3 \end{bmatrix}_{3 \times 3}$$

3) Diagonal Matrix \rightarrow In this ~~element~~ matrix all elements are zero except diagonal elements. We can say all non-diagonal elements are zero.

Ex.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

4) Scalar Matrix \rightarrow In which all diagonal elements are equal to each other. Means $a_{11} = a_{22} = a_{33}$

Ex.

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

5] Identity/Unity Matrix → In which ~~are~~ all diagonal elements are equal to unity i.e., 1
Exp.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6] Row Matrix → A matrix which has only one row and any number of columns is called row matrix.

Exp.

$$A = \begin{matrix} & C_1 & C_2 & C_3 \\ \begin{bmatrix} 3 & 1 & 2 \end{bmatrix} & R_1 & & \end{matrix}, B = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \begin{bmatrix} 3 & 3 & 1 & 4 & 5 \end{bmatrix} & R_1 & & & \end{matrix}$$

7] Column Matrix → which matrix has only one column and any number of rows can be there.

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, B = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

8] Null Matrix → A matrix whose all elements are equal to zero.

Exp.

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(9) Symmetric Matrix \rightarrow If the matrix & its transpose are equal, then matrix is symmetric matrix.
Ex.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 6 & 4 \\ 5 & 4 & 7 \end{bmatrix}_{3 \times 3}$$

then $A' = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 6 & 4 \\ 5 & 4 & 7 \end{bmatrix}_{3 \times 3}$

(10) Skew-Symmetric Matrix \rightarrow A square matrix is called a skew symmetric matrix if $a_{ij} = -a_{ji}$ for all values of i & j . It means values of i & j elements are equal to the negative of (j, i) . In simple words.

$$a_{12} = -a_{21}$$

$$a_{13} = -a_{31}$$

$$a_{23} = -a_{32}$$

It means that every diagonal element of skew-symmetric matrix is zero.

$$a_{11} = 0, a_{22} = 0, a_{33} = 0$$

$$a_{ij} = 0, a_{ji} = 0$$

(ii) Orthogonal Matrix \rightarrow A square matrix is said to be orthogonal matrix when the product of transpose of the Matrix A with the original matrix is equal to unity i.e. I

This a matrix is orthogonal when

$$A' A = I$$

or

$$A' = A^{-1}$$

(2) \rightarrow Addition of Matrix \rightarrow

~~Matrices can be added or subtracted if and only if they~~
 Addition of two or more matrices is possible only if when the ~~same~~ order of the matrices is same.

It means if two or more matrices are of same order then their sum can be obtained by adding of their respective elements.

Ex.

$$A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}, B = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}_{2 \times 2}$$

then

$$A + B = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4+3 & 2+1 \\ 3+2 & 4+3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ 5 & 7 \end{bmatrix}$$

→ Subtraction of Matrix → Subtraction of two or more

matrices is possible only when the order of the matrices is same. In other words if the two or more matrices are of the same order then their difference can be obtained by subtracting their respective elements.

Ex.

$$A = \begin{bmatrix} 7 & 5 & 3 \\ 6 & 9 & 6 \\ 8 & 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 & 1 \\ 3 & 3 & 4 \\ 5 & 2 & 2 \end{bmatrix}$$

Now

$$\therefore A - B = \begin{bmatrix} 7 & 5 & 3 \\ 6 & 9 & 6 \\ 8 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 1 \\ 3 & 3 & 4 \\ 5 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7-4 & 5-1 & 3-1 \\ 6-3 & 9-3 & 6-4 \\ 8-5 & 4-2 & 2-2 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 3 & 4 & 2 \\ 3 & 6 & 2 \\ 3 & 2 & 0 \end{bmatrix} \quad \underline{\underline{\text{Ans}}}$$

→ Multiplication of Matrices → Two matrices can be multiplied if and only if number of column in first matrix is equal to number of ^{matrix} column of ~~the~~ rows in second ~~column~~.

$$\text{Ex.} \rightarrow A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 \times 5 + 2 \times 7 & 1 \times 6 + 2 \times 8 \\ 3 \times 5 + 4 \times 7 & 3 \times 6 + 4 \times 8 \end{bmatrix}$$

$$= \begin{bmatrix} 5 + 14 & 6 + 16 \\ 15 + 28 & 18 + 32 \end{bmatrix}$$

$$AB = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \quad \underline{\underline{Ans}}$$